

Superquintessence

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Abstract

There is marginal evidence that the quintessential form of matter responsible for the acceleration of the universe observed today has ratio between pressure and energy density $w < -1$. Such a regime, called superacceleration, cannot be achieved with conventional scalar field models. The simplest non-exotic model achieving superacceleration is that of a scalar field nonminimally coupled to the Ricci curvature. This model is studied for general potentials and an exact superaccelerating solution is presented. In quintessential inflation, the model can have blue gravitational wave spectra, improving the prospects for the detection of cosmological gravitational waves.

1 Introduction

Observations of the magnitude-redshift relation of distant Type Ia supernovae [1] led to the recent discovery that the universe undergoes accelerated expansion at the present era. This acceleration regime is attributed to the influence of a yet unknown form of non-luminous matter called *quintessence*, which must have negative pressure [2].

Several theoretical models of quintessence have been proposed, including a cosmological constant, a time-varying energy density and, more plausibly, a scalar field [2].

There is no shortage of scalar fields in modern particle physics and in gravitational theories; it suffices to think of the dilaton of string theories [3], of the supersymmetric partner of spin 1/2 particles in supergravity, of Nambu-Goldstone bosons, of the Higgs field of the standard model, or of the inflaton field dominating the cosmic dynamics at early times [4], the scalar of geometric origin of Kaluza-Klein theories [5], or the Brans-Dicke scalar [6]. It is not surprising, therefore, that scalar fields have been widely employed as natural models of quintessence.

An accelerated expansion of the universe is obtained when the pressure P and the energy density ρ of the scalar field satisfy $w \equiv P/\rho < -1/3$. On the observational side, Ref. [7] pointed out that there is marginal evidence for values of the w -parameter less than -1 . This range of values corresponds to a superexponential expansion that we call *superacceleration*. An ordinary scalar field minimally coupled to the spacetime curvature, used in most models, cannot achieve such a regime (this will be shown in detail in Sec. 2); Ref. [7] therefore spurred interest in non-conventional, supergravity-inspired, scalar field models of quintessence exhibiting the “wrong” sign of the kinetic energy density [8, 9, 10, 11].

In this paper it is demonstrated that a superacceleration regime can be achieved in a much simpler model containing only a scalar field nonminimally coupled to the Ricci curvature and with positive definite kinetic energy density. Such a model is not only esthetically appealing by virtue of simplicity, but is also rather compelling from a theoretical point of view, since nonminimal coupling necessarily arises due to first loop corrections (see Ref. [12] for a recent review and references), or as a prediction of specific scalar field theories [13, 14]; and even at the classical level, nonminimal coupling is required by the Einstein equivalence principle in general relativity [15].

The dynamics of a scalar field nonminimally coupled to gravity were recently investigated in a general dynamical system approach [16, 17, 18, 19, 20] and in the context of inflation [21, 12, 22]. Several results scattered in these references are relevant for quintessence and, in particular, for the superacceleration regime. In the present work, the relevant aspects of this approach for superacceleration are collected and discussed,

while new features are pointed out. Instead of assuming a specific form of the scalar field potential $V(\phi)$, as done in previous models of quintessence based on nonminimally coupled scalars [23], we keep the form of $V(\phi)$ completely general.

Theoretical evidence for superacceleration in nonminimally coupled models is pointed out, and an exact solution is derived which is superaccelerating and has an effective equation of state described by a time-dependent ratio $w(t) = P/\rho$.

The implications of present-day superacceleration (if confirmed) for quintessential inflationary scenarios of the universe are pointed out. Contrarily to minimally coupled scalar field models, blue gravitational wave spectra are possible; the resulting higher power at small scales makes nonminimally coupled scalar field models interesting from the point of view of the detection of primordial gravitational waves with present and future laser interferometers.

Certain features of the dynamics of nonminimally coupled scalars also occur in string-inspired pre-big bang cosmology [24]. The latter was criticized on the basis of being unable to provide a true inflationary regime - the ratio of any physical length to the Planck length scale, that is a true measure of inflation, actually decreases in pre-big bang cosmology, thus failing to solve the horizon problem [25]. A similar problem is present in any dilaton-driven regime of cosmic acceleration, but is absent in the theory presented in this paper.

The plan of the paper is as follows: in Sec. 2 the effective equation of state of a universe dominated by a scalar field is discussed, and it is shown that superacceleration is absent in a minimally coupled scalar field model. The simplest model of superquintessence (the material source fueling the yet unconfirmed, present-day, superacceleration regime of the universe) is then introduced, and the relevant equations are derived. In Sec. 3, an exact superaccelerating solution is presented: Sec. 4 discusses the implications for quintessential inflationary models, and the gravitational wave spectra constituting a distinctive feature of superaccelerated cosmic expansion, and contains outlooks on nonminimally coupled scalar field models of quintessence.

2 Superquintessence and superacceleration

It was pointed out in Ref. [7] that models with $w \equiv P/\rho < -1$ are in agreement with observations of Type Ia supernovae [1], even for very negative w , and there is marginal evidence that the pressure to density ratio is indeed less than -1 . Certainly, the $w < -1$ parameter space is not excluded by the available observational data [1]. The possibility that $w < -1$ was investigated by several authors [7, 8, 9, 10, 11] mainly

using models based on a scalar field with the “wrong” sign of the kinetic energy term in the Lagrangian. Dark energy density with the property $w < -1$ was called *phantom energy* by these authors.

The search for a rather exotic model of phantom energy is justified by the following considerations: $P < -\rho$ is equivalent to $\dot{H} > 0$, where H is the Hubble parameter and an overdot denotes differentiation with respect to the cosmic time, and we refer to a spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe strongly favoured by cosmological observations. The spacetime metric is [26]

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) \quad (2.1)$$

in comoving coordinates (t, x, y, z) , and $H = \dot{a}/a$ satisfies the equations [4]

$$H^2 = \frac{\kappa}{3} \rho, \quad (2.2)$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{\kappa}{6} (\rho + 3P), \quad (2.3)$$

where $\kappa \equiv 8\pi G$ and ρ and P are, respectively, the energy density and pressure of the cosmic fluid. Eqs. (2.2) and (2.3) imply that

$$\dot{H} = -\frac{\kappa}{2} (\rho + P) \quad (2.4)$$

and therefore

$$P < -\rho \Leftrightarrow w < -1 \Leftrightarrow \dot{H} > 0. \quad (2.5)$$

A regime with $\dot{H} > 0$ was originally investigated in the context of inflationary models of the early universe and called *superinflation* [28], a name later adopted in string cosmology. Perhaps a better name would be *superacceleration*, to denote the possibility that $\dot{H} > 0$ in a quintessence-dominated universe today, well after the end of inflation; we use this terminology throughout the present paper.

The simplest models of quintessence and inflation are based on a scalar field minimally coupled to gravity and are not adequate to describe a superacceleration regime; in fact, in such models, the scalar field $\phi(t)$ behaves as a perfect fluid with energy density and pressure

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \quad (2.6)$$

$$P = \frac{\dot{\phi}^2}{2} - V(\phi), \quad (2.7)$$

respectively, where $V(\phi)$ is the scalar field potential. Eq. (2.4) becomes, in this case,

$$\dot{H} = -\frac{\kappa}{2}\dot{\phi}^2. \quad (2.8)$$

Therefore, $\dot{H} \leq 0$ in such models, with $\dot{H} = 0$ corresponding to a de Sitter solution with scale factor $a(t) = a_0 e^{Ht}$. The effective equation of state of a FLRW universe dominated by a minimally coupled scalar field is given, for a *general* potential $V(\phi)$, by

$$\frac{P}{\rho} = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V} \equiv w(x), \quad (2.9)$$

where $x \equiv \dot{\phi}^2/2V$ is the ratio between the kinetic and the potential energy densities of the scalar ϕ . Under the usual assumption $V \geq 0$ guaranteeing that the energy density of ϕ is non-negative, the function

$$w(x) = \frac{x - 1}{x + 1} \quad (2.10)$$

monotonically increases from its minimum $w_{min} = -1$ attained at $x = 0$ to the horizontal asymptote $+1$ as $x \rightarrow +\infty$ (corresponding to $V = 0$). The slow-rollover regime of inflationary models corresponds to $x \ll 1$ and to $w(x)$ near its minimum, where the kinetic energy density $\dot{\phi}^2/2$ of ϕ is negligible in comparison to its potential energy density $V(\phi)$. As the kinetic energy density increases, the equation of state progressively deviates from $P = -\rho$ and the pressure becomes less and less negative. The system gradually moves away from the slow-rollover regime. At equipartition between the kinetic and the potential energy densities ($x = 1$), one has the dust equation of state $P = 0$. The pressure then becomes positive as x increases and, when the kinetic energy density completely dominates the potential energy density ($x \gg 1$), one finally reaches the stiff equation of state $P = \rho$. Thus, for minimally coupled scalar fields, one encompasses the range of equations of state

$$-1 \leq w \leq 1 \quad (2.11)$$

and observational data producing $w < -1$ are not explained by this “canonical” scalar field model, unless one is willing to accept a negative potential $V(\phi) < -\dot{\phi}^2/2$, which yields the negative energy density $\rho = \dot{\phi}^2/2 + V(\phi)$. This violation of the weak energy condition is very unappealing, and the possibility that $V < 0$ is excluded from the rest of this work.

The superacceleration regimes studied in the literature consist of pole-like inflation with scale factor

$$a(t) = \frac{a_0}{t - t_0} , \quad (2.12)$$

a special form of superacceleration considered in early inflationary theories [28], in pre-big bang cosmology [24], and in Brans-Dicke theory [25].

Situations with $P < -\rho$ can also occur in higher derivative theories of gravity [29] or due to semiclassical particle production resulting in nonzero bulk viscosity [30]. In the words of Ref. [7], the marginal evidence for phantom energy with $w < -1$ (which we prefer to call *superquintessence*, consistently with the word “superacceleration”) poses the challenge of building a microphysical model of phantom energy (superquintessence). Rather unconventional, supergravity-inspired models with kinetic energy density of the scalar field $-\dot{\phi}^2/2$ instead of $\dot{\phi}^2/2$ were investigated ([7, 8, 9, 10]-see also [31]). When the scalar field potential V is absent, this form of matter is called *kinetically driven quintessence*. While there certainly is scope for investigating such models, Occam’s razor dictates that one should first study the simplest, most natural model of superquintessence; this is the very simple theory of a scalar field nonminimally coupled to gravity, described by the action

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla^c \phi \nabla_c \phi - V(\phi) - \frac{\xi}{2} R \phi^2 \right] , \quad (2.13)$$

where g is the determinant of the metric tensor and the kinetic energy density term is canonical, hence the latter is positive definite in a FLRW space. We do not include other forms of matter in the action, as it would be necessary to build a complete model of quintessence. We assume that the quintessential field ϕ has already begun to dominate the dynamics of the universe. That this scenario is plausible, and that the cosmic coincidence problem can be solved in this context, was already shown in previous works [23]. Variation of the action with respect to g_{ab} leads to the field equations

$$G_{ab} = \kappa T_{ab} , \quad (2.14)$$

where the scalar field energy-momentum tensor is

$$T_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi - V g_{ab} + \xi (g_{ab} \square - \nabla_a \nabla_b) (\phi^2) + \xi G_{ab} \phi^2 . \quad (2.15)$$

Note that, in the presence of nonminimal coupling (hereafter referred to as “NMC”), there are three possible inequivalent ways of writing the field equations (see Refs. [22, 32]

for a recent discussion) and the scalar field stress-energy tensor. We choose the procedure described by eqs. (2.14) and (2.15) because

i) the corresponding energy density

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi) + 3\xi H\dot{\phi} (H\phi + 2\dot{\phi}) \quad (2.16)$$

is always positive definite, since it is related to the Hubble parameter by the Hamiltonian constraint

$$H^2 = \frac{\kappa}{3} \rho , \quad (2.17)$$

which guarantees that $\rho \geq 0$. This is not the case with other definitions of energy density used in the literature, which has led to debate the validity of the weak energy condition [33, 32, 34] for nonminimally coupled classical scalars.

ii) The gravitational coupling is constant in the approach of this paper, while the effective gravitational coupling

$$\kappa_{eff} = \frac{\kappa}{1 - \kappa\xi\phi^2} \quad (2.18)$$

used in other approaches and widely present in the literature can change sign and diverge, leading to spurious effects and loss of generality in the field equations when the scalar field attains the critical values

$$\pm \phi_c = \pm \frac{1}{\sqrt{\kappa\xi}} \quad (2.19)$$

for $\xi > 0$ (for examples see Refs. [19, 33]). The nonminimally coupled scalar field has pressure given by

$$P = \frac{\dot{\phi}^2}{2} - V(\phi) - \xi [4H\phi\dot{\phi} + 2\dot{\phi}^2 + 2\phi\ddot{\phi} + (2\dot{H} + 3H^2)\phi^2] \quad (2.20)$$

and obeys the equations

$$6 [1 - \xi (1 - 6\xi) \kappa\phi^2] (\dot{H} + 2H^2) - \kappa (6\xi - 1) \dot{\phi}^2 - 4\kappa V + 6\kappa\xi\phi V' = 0 , \quad (2.21)$$

$$\frac{\kappa}{2} \dot{\phi}^2 + 6\xi\kappa H\phi\dot{\phi} - 3H^2 (1 - \kappa\xi\phi^2) + \kappa V = 0 , \quad (2.22)$$

$$\ddot{\phi} + 3H\dot{\phi} + \xi R\phi + V' = 0 . \quad (2.23)$$

The fact that a nonminimally coupled scalar field can generate a superacceleration regime was already known to the authors of Ref. [28] in the context of inflation (specifically,

pole-like inflation), and was recently revisited in Refs. [17, 18]; a dynamical systems approach clearly showed the possibility of superacceleration and numerical solutions with $\dot{H} > 0$ were presented. It was speculated that such a regime could be important for the possible genesis of the universe from Minkowski space and for the modelling of quintessence. Further, in Ref. [20], an exact solution corresponding to integrability of the field equations and to the equation of state $P = -5\rho/3$ was found.

The fundamental differences between our discussion and previous ones are that we do not impose that the ratio $w = P/\rho$ be constant, and we do not choose a specific form of $V(\phi)$, but rather keep the discussion valid for *any* potential. Quintessence scenarios with a specific potential and NMC, as well as generalized Brans-Dicke couplings, or induced gravity, can be found in the literature, although superacceleration was not studied there [23, 35].

3 An exact superaccelerating solution

The assumption $w = \text{const.}$ used in the literature is useful to find analytical solutions, but is very restrictive: by imposing $w = \text{const.}$ in a spatially flat FLRW universe one can only obtain power-law ($a = a_0 t^p$) and de Sitter solutions (this is true for both minimal and nonminimal coupling), or a third exact solution presented in Ref. [36]. Solutions for non-spatially flat universes and arbitrary values of w are summarized in [37]. The search for analytical solutions probably constitutes the reason why only pole-like inflation (an inverse power-law) was discussed in the literature in conjunction with superacceleration [28, 25, 24]. Superaccelerating solutions are more easily found numerically [17, 18].

A new explicit solution of the field equations (2.21)-(2.23) is obtained by setting ϕ equal to one of the critical values $\pm\phi_c$ given by eq. (2.19); then the trace of the field equations

$$R = 6(\dot{H} + H^2) = \kappa(\rho - 3P) \quad (3.1)$$

yields

$$\dot{H} + 2H^2 = C, \quad (3.2)$$

where C is a constant. For $C > 0$ one has

$$\frac{\dot{H}}{1 - 2H^2/C} = C \quad (3.3)$$

which is reduced to a quadrature as

$$\int \frac{dx}{1 - x^2} = \sqrt{2C}(t - t_0), \quad (3.4)$$

where $x \equiv \sqrt{2/C} H$. One has

$$\operatorname{arctanh} x = \sqrt{2C} (t - t_0) \quad (3.5)$$

if $x^2 > 1$ and

$$\operatorname{arctanh} x = \ln \left[\sqrt{\frac{1+x}{1-x}} \right] \quad (3.6)$$

if $x^2 < 1$, and therefore

$$H = \sqrt{\frac{C}{2}} \tanh \left[\sqrt{2C} (t - t_0) \right] \quad (3.7)$$

in both cases $H > \sqrt{C/2}$ and $H < \sqrt{C/2}$ (the cases $H = \pm \sqrt{C/2}$ correspond to trivial de Sitter solutions). Eq. (3.7) contradicts the limit $H > \sqrt{C/2}$ and hence (3.7) is only a solution for $H < \sqrt{C/2}$, corresponding to the scale factor

$$a = a_0 \cosh^{1/2} \left[\sqrt{2C} (t - t_0) \right] . \quad (3.8)$$

This solution describes an asymptotic contracting de Sitter space as $t \rightarrow -\infty$, reaching a minimum size at $t = t_0$, and then expanding and superaccelerating with $\dot{H} > 0$ and $w < -1$. As $t \rightarrow +\infty$, the solution approaches an expanding de Sitter space. The effective equation of state is time-dependent, with

$$w(t) = \frac{P}{\rho} = - \frac{A + \xi (2\dot{H} + 3H^2)}{3H^2 + A} = C \left\{ 2 - \frac{1}{2} \tanh^2 \left[\sqrt{2C} (t - t_0) \right] \right\} , \quad (3.9)$$

where $A = \kappa V(\pm\phi_c)$. While this exact solution is clearly fine-tuned in the scalar field value, it has the merit of explicitly illustrating the superacceleration phenomenon; the latter is a generic feature of the dynamics of a nonminimally coupled scalar field. This was shown by considering the natural potential

$$V(\phi) = \frac{m^2}{2} \phi^2 + \lambda \phi^4 \quad (3.10)$$

and the conformal case $\xi = 1/6$ (the value of ξ that is an infrared fixed point of the renormalization group [38, 13] and is required by the Einstein equivalence principle [15]). The dynamics of this system are much richer than in the minimally coupled ($\xi = 0$) case and were studied in detail [19, 17, 18, 16].

4 Outlooks

If the nonminimally coupled quintessence field was also responsible for inflation early in the history of the universe, as in certain quintessential inflationary scenarios [39], one has to verify that the slow-roll approximation necessary to build a general theory of inflation is meaningful. In other words, one should check that the de Sitter solutions behaving as attractor points in phase space when $\xi = 0$, remain attractors when NMC is added to the theory. This is true subject to certain constraints on the value of the coupling constant ξ and the potential $V(\phi)$ [21, 12]. The following dimensionless slow-roll parameters can be introduced [40, 12]

$$\epsilon_1 = \frac{\dot{H}}{H^2} , \quad (4.1)$$

$$\epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}} , \quad (4.2)$$

$$\epsilon_3 = - \frac{\xi \kappa \phi \dot{\phi}}{H \left[1 - \left(\frac{\phi}{\phi_1} \right)^2 \right]} , \quad (4.3)$$

$$\epsilon_4 = - \frac{\xi (1 - 6\xi) \kappa \phi \dot{\phi}}{H \left[1 - \left(\frac{\phi}{\phi_2} \right)^2 \right]} , \quad (4.4)$$

where

$$\pm \phi_2 \equiv \frac{\pm 1}{\sqrt{\kappa \xi (1 - 6\xi)}} \quad (4.5)$$

for $0 < \xi < 1/6$. ϵ_3 and ϵ_4 vanish in the limit $\xi \rightarrow 0$ of ordinary inflation; ϵ_4 also vanishes for conformal coupling ($\xi = 1/6$). One has $|\epsilon_i| < 1$ ($i = 1, 2, 3, 4$) for every solution attracted by expanding de Sitter spaces (H_0, ϕ_0) for suitable values of ξ and V [21, 12]. Moreover, $\epsilon_i = 0$ exactly for de Sitter solutions with constant scalar field [41].

The spectral indices of scalar and tensor perturbations [4] are expressed in terms of the slow-roll parameters [40, 12] by

$$n_S = 1 + 2(2\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4) , \quad (4.6)$$

$$n_T = 2(2\epsilon_1 - \epsilon_3) . \quad (4.7)$$

In a superacceleration regime during inflation, with $\dot{H} > 0$, one has $\epsilon_1 > 0$ and the spectral index of gravitational waves n_T given by eq. (4.7) can be positive (*blue spectrum*),

with more power at small wavelengths than the usual inflationary perturbation spectra. This feature is very attractive for the gravitational wave community because it enhances the possibility of detection of cosmological gravitational waves. Detailed calculations for specific potentials (beyond the purpose of this work) are required in order to assess the resulting gravitational wave amplitudes in the frequency bands covered by the *LIGO*, *VIRGO*, and the other present laser interferometry experiments; it seems that blue spectra are meaningful at least for the future space-based interferometers [42]. Blue spectra are impossible with minimal coupling [43], for which one obtains $n_T = 4H/H^2 \leq 0$.

Another interesting feature emerging from the studies of Refs. [17, 18] is the *spontaneous* exit of the universe from the superacceleration regime, to enter an ordinary accelerated one, followed again by spontaneous exit and entry into a decelerated epoch. These features are absent in string-based models that keep accelerating forever [44, 45], or in inflationary models where inflation is terminated by an *ad hoc* modification of the equations or of the shape of $V(\phi)$.

Finally, we remark that the $\dot{H} > 0$ regime achieved with NMC is a true superacceleration regime, contrarily to the case of pre-big bang cosmology and of Brans-Dicke theory. As pointed out in Ref. [25] in these theories, while the universe expands, the Planck length also grows and the ratio of any physical length to the Planck length (which is a true measure of the inflation of the universe) actually *decreases*. In string-inspired pre-big bang cosmology this phenomenon is due to the fact that the gravitational coupling is $G e^\Phi$, where $\Phi(t)$ is the string dilaton [25].

For a nonminimally coupled scalar field, using the approach selected here and the corresponding T_{ab} for the scalar field, the gravitational coupling is constant and there is no variation of the Planck length; the superacceleration regime is a true one. A different approach using the effective gravitational coupling $G_{eff} = G(1 - 8\pi G\xi\phi^2)^{-2}$ common in the literature, would suffer of the same problem of pre-big bang cosmology for $\xi > 0$ and could not produce the exact solution (3.8) since G_{eff} diverges for the corresponding scalar field values $\pm\phi_c$.

Were the inequality $w < -1$ supported by future observations of distant supernovae, the conventional quintessence models based on minimally coupled scalars would have to be abandoned in favour of alternative models: among these, the nonminimally coupled theory here described appears as the simplest and most promising.

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